LORENTZ TRANSFORMATIONS ARE UNABLE TO DESCRIBE THE RELATIVISTIC DOPPLER EFFECT

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Communicated by Dr. M. Iqbal Choudhary

Abstract: We know paradoxes do not exist in nature and a complete theory does not include them. We only need to use our intuition to see that there are no logical inconsistencies. In this context, the relationship between the Relativistic Doppler Effect (RDE) and Lorentz Transformations (LT) exhibits logical inconsistencies. It means that LT misrepresents reality and describes no physical effects. In this paper we will explain how to eliminate such logical inconsistencies (contradictions).

Keywords: Doppler effect, speed of light, time dilation, special relativity theory

Introduction

In his well known article on "Special Relativity" Einstein succeeded in deriving LT [1], and then deriving the relativistic Doppler relations based on these transformations. Therefore relativistic Doppler effect (RDE) is related to the time dilation effect [2]. SRT accounts for various kinematical effects, like length contraction and time dilation. Several questions arise when examining this kinematical effect and many contradictions exist. Moreover SRT and relativistic Doppler relations are incompatible. In this regard we will see that the asymmetry of kinematical time dilation effect derived by LT makes it difficult to reconcile LT effects and RDE completely.

In this paper, we depend on the method in [3,4], in which it is shown that the Doppler calculation procedure as well as interpretation is possible only with the help of the Lorentz force law and the relativity principle. This will end the role of the Lorentz transformation (LT) and of time dilation in RDE.

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Einstein's Method in Deriving Doppler's Formula

It is well known that the color of light rays coming out of a moving source towards the observer tend to be blue shifted (i.e. high frequency), whereas a ray exiting a moving source in the opposite direction tends to be red shifted (i.e. low frequency). The diversity of possibilities along with the existence of ether between the source and the observer leads to four possibilities, explained as follows.

1. If the source is receding/approaching from the rest observer, the frequency that observer sees is classically1.

$$f'=\frac{f_0}{1+(u/c)}, \qquad f'=\frac{f_0}{1-(u/c)} \qquad (1a,1b)$$

2. If the observer is receding/approaching from the rest source, the frequency that observer sees is classically

$$f'=f_0\left(1-\frac{u}{c}\right), \qquad f'=f_0\left(1+\frac{u}{c}\right) \qquad (2a,2b)$$

By excluding the idea of ether, Einstein has reduced these four possibilities to only two, namely,

$$f' = \sqrt{1 - \frac{u^2}{c^2}}\, \frac{f_0}{1 + \frac{u}{c}} = f_0 \sqrt{\frac{1 - u/c}{1 + u/c}} \qquad (3a)$$

$$f' = \sqrt{1 - \frac{u^2}{c^2}}\, \frac{f_0}{1 - \frac{u}{c}} = f_0 \sqrt{\frac{1 + u/c}{1 - u/c}} \qquad (3b)$$

Relativity principle makes it easier to use when it considers these two possibilities as actually one single possibility, and we get the second possibility through converting the speed sign in the first.

We can see the difference between the classical Doppler effect applied to light waves and RDE. It makes no sense to talk about the velocity of either the source or the observer relative to the medium as one does in ether. One considers only the relative velocity u between the source and the observer as the RDE includes time dilation, i.e. the RDE includes also the transverse Doppler effect (TDE).

Einstein obtained in his work [1] the following formula:

$$f' = f_0\, \frac{\sqrt{1 - (u^2/c^2)}}{1 + (u/c)\cos\theta} \qquad (4)$$

where u is the speed of source,   is the direction of travel.

There are two particular cases that lead to simplifications. The first is motion along the line of sight-the longitudinal relativistic Doppler effect, where

1- $$f' = \sqrt{1 - \frac{u^2}{c^2}}\, \frac{f_0}{1 + \frac{u}{c}} = f_0 \sqrt{\frac{1 - u/c}{1 + u/c}} \qquad (5a)$$

Had the source been approaching from the observer, then

2- $$f' = \sqrt{1 - \frac{u^2}{c^2}}\, \frac{f_0}{1 - \frac{u}{c}} = f_0 \sqrt{\frac{1 + u/c}{1 - u/c}} \qquad (5b)$$

The other special case is that of transverse motion across the line of sight. In this case

3- $$f' = f_0 \sqrt{1 - \frac{u^2}{c^2}} = \frac{f_0}{\gamma} \qquad (5c)$$

Eqs. (5a, 5b) have classical analogues in Eqs. (1a, 1b).

The frequency is red shifted due to the dilation of the source time, Eq. (5c), and this effect (TDE) corresponds to the time slowing down on the source moving clock.

In SRT's formalism the key effect for RDE is time dilation, which plays an important part in modern physics. Therefore Eq. (5c) is considered a unique feature of SRT and is related to the dilation of time only for the moving source. Therefore a statement was raised in textbooks that "The TDE should not occur in classical physics". This statement is not correct in some models [5,6], and it is not correct if one derives TDE without ether theory or any relativistic assumption such as time dilation as we will do in the present paper.

Einstein believed that the general formula Eq. (4) which he deduced is an appropriate

formula for the two cases, the source is moving and observer is at rest, or the source is at rest and the observer is moving. As remarked in Einstein's method, RDE and TRD modes treat only source receding/approaching from the rest observer. However, an accurate analysis of Eq. (4) by using LT would reveal that there is an important contradiction between RDE and LT.

According to the relativity principle, Eq. (4) could be written for the case of observer in motion as,

$$f' = f_0 \, \gamma (1 - \frac{u}{c} \cos \theta)$$

If the motion is normal to the line connecting source and observer, we then obtain from the last equation

$$f' = \gamma \, f_0$$

This equation shows a time contraction, instead of a time dilation as in Eq. (5c). We know that time contraction does not exist in SRT but the symmetry effect of TDR requires a time contraction.

We will see now that the asymmetry of kinematical time dilation effect derived by LT makes it difficult to reconcile LT effects and RDE completely.

Derivation of RDE and TDE from Lorentz Transformations

Assume two inertial frames S and S', a source with frequency f_0 in the moving frame , an observer in the rest frame , and the source approaching with the relative velocity $u \parallel ox$ from the observer.

The position of the radiation frequency of moving source is described by

$$x = ct, \qquad x' = ct' \tag{6}$$

where $(x, \, t)$ and (x',t') are spatial and time intervals. Then, we apply LT

where $(x, \, t)$ and $(x', \, t')$ are spatial and time intervals. Then, we apply LT

$$x' = \gamma \, (x - u t), \ \ t' = \gamma \, (t - \frac{u}{c^2} x) \tag{7}$$

we have

$$t' = \gamma \, t \, (1 - \frac{u}{c}) \tag{8}$$

The frequency is derived as the inverse of time, i.e.

$$t = \frac{1}{f_0}, \qquad t' = \frac{1}{f'} \tag{9}$$

If we insert Eq. (9) in (8), we find

$$f' = \frac{f_0 \, \sqrt{1 - (u^2/c^2)}}{1 - (u/c)}$$

$$= f_0 \, \sqrt{\frac{1 + (u/c)}{1 - (u/c)}} \tag{10}$$

Had the source been receding from the observer, then by replacing u with $-u$ in Eq. (17), we have

$$f' = f_0 \, \sqrt{\frac{1 - (u/c)}{1 + (u/c)}} \tag{11}$$

Let the reference time in S be t_0. The reference time in S' is defined by Eq. (7), i.e.,

$$t' = \gamma \, t_0 \tag{12}$$

Using (9) in (12), we get

$$f' = \frac{f_0}{\gamma} = f_0 \sqrt{1-(u^2/c^2)} \qquad (13)$$

Eqs. (10,11 and 13) are the relativistic Doppler shift derived by Einstein [1], but now is derived from LT directly. According to relativity principle, we can also consider the frame to be co-moving with the source and receding/ approaching the observer. Then Eq. (7) could be written as

$$x = \gamma(x' + u t') , \quad t = \gamma(t' + \frac{u}{c^2} x') \qquad (14)$$

Inserting (6) in (14), we obtain

$$t = \gamma t' (1 + \frac{u}{c})$$

Then using (9) in the last equation, we have

$$f_0 = \frac{f' \sqrt{1-(u^2/c^2)}}{(1+\frac{u}{c})}$$

or

$$f' = \frac{f_0 (1 + \frac{u}{c})}{\sqrt{1 - (u^2/c^2)}}$$

$$= f_0 \sqrt{\frac{1 + u/c}{1 - u/c}} \qquad (15)$$

Had the observer been receding from the source, then by replacing u with $-u$ in Eq. (15), we have

$$f' = f_0 \sqrt{\frac{1 - (u/c)}{1 + (u/c)}} \qquad (16)$$

If the motion is normal to the line connecting source and observer, we then obtain

$$f' = \frac{f_0}{\sqrt{1 - \frac{u_T^2}{c^2}}} = \gamma \, f_0 \qquad (17)$$

where in normal movement, the radial component is zero, $u_r = 0$, and since

$$u^2 = u_t^2 + u_r^2 = u_t^2$$

we obtain also a change in frequency as per Eqs. (17). So, the Doppler effect exists even though there is no component of relative motion along the line of sight. The reference time of the moving observer becomes long, i.e. according to Eqs. (9) and (17), we have

$$t' = t_0 / \gamma \qquad (18)$$

The reference time of the moving observer decreases (time contraction). Thus the frequency of the light source that is seen by the moving observer increases as in Eq. (15) and decreases as in Eq. (16).

Eqs. (15,16 and 17) were not derived by Einstein since the case of rest source and moving observer require time contraction as in Eq.(18) while the kinematical effect derived by LT is time dilation and not time contraction. Thus, the Lorentz transformation (7) and (14) have been used for the calculation of the so called relativistic effect, and we now know that they give the possibility to calculate the relativistic effect only if LT has time contraction as in Eq. (18). This means that LT misrepresents reality and reflects no physical effects [3,4].

Derivation of RDE and TDE from Lorentz Force

Now assume that a particle q in frame S as ion (source) emits a light wave (photon) that

moves in a direction that makes an angle θ with the positive ox axis. The light is received at the observer in frames S' at an angle θ' relative to the ox' axis. In [3,4] we have derived the following relation:

$$v' = \gamma v(1 - \frac{u}{c}\cos\theta) \qquad (19a)$$

But the connection between the two frequencies in frames S and is S' given also by

$$v = \gamma v'(1 + \frac{u}{c}\cos\theta') \qquad (19b)$$

If we consider the frame S to be co-moving with the source and receding /approaching observer, Eq. (19a) becomes

$$1\text{-}\theta = \theta' = 0° \text{ i.e. } v' = v_0\gamma(1-\frac{u}{c}) = v_0\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} \qquad (20a)$$

$$2\text{-}\theta = \theta' = 180° \text{ i.e. } v' = v_0\gamma(1+\frac{u}{c}) = v_0\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}} \qquad (20b)$$

Eqs. (20a,20b) do not have relativistic analogues, but have classical analogues in Eqs. (2a,2c). According to relativity principle, we can also consider the frame to be co-moving with the observer and receding /approaching source, then Eq. (19b) could be written as

$$v' = \frac{v_0\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}\cos\theta'} \qquad (20c)$$

Hence

$$1\ \theta = \theta' = 0° \text{ i.e. } v' = \frac{v_0\sqrt{1-\frac{u^2}{c^2}}}{1+\frac{u}{c}} = v_0\sqrt{\frac{1-\frac{u}{c}}{1+\frac{u}{c}}} \qquad (21a)$$

$$2\text{-}\theta = \theta' = 180° \text{ i.e. } v' = \frac{v_0\sqrt{1-\frac{u^2}{c^2}}}{1-\frac{u}{c}} = v_0\sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}} \qquad (21b)$$

Eqs. (21a, 21b) are identical to Eqs. (5a, 5b) in SRT, and the classical analogues are (1a, 1b).

If the velocity of the observer/source is perpendicular to the line of sight, then we have from Eq. (19a),

$$\theta = \theta' = 90° \text{ i.e. } v' = \frac{v_0}{\sqrt{1-\frac{u^2}{c^2}}} = \gamma v_0 \qquad (22a)$$

and from Eq. (20c), we have

$$\theta = \theta' = 90° \text{ i.e. } v' = v_0\sqrt{1-\frac{u^2}{c^2}} = \frac{v_0}{\gamma} \qquad (22b)$$

Formula (22a) has been confirmed in a Mossbauer experiment with a moving detector [7], and formula (22b) was early verified by Ives and Stilwell experiments [8,9].

In [3] we have shown, that had it not been for the existence of formula (22a) and (22b) together, there would be no equality between the two formulas (20a) and (21a), and the two formulas (20b) and (21b). This means that formula (22a) exists since this formula is not the outcome of SRT due to time contraction. This

would mean that LT is unable to describe a well known physical reality, namely the Doppler effect.

We turn to Eqs. (19a,20c) i.e.

$$v' = \mathcal{W}_0(1 - \frac{u}{c}\cos\theta) \text{ ,and } v' = \frac{v_0\sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u}{c}\cos\theta'} \quad (23)$$

Einstein has used LT to give a new interpretation of the astronomically observed phenomenon of aberration. This was indeed already discovered in the early eighteenth century by Bradley [10]. He gave an explanation on the basis of the ether theory and the finite velocity of light. Hence, in this paper the explanation did not have to wait for the construction of LT. The two angles in Eq. (23) differ when we include the effect of aberration. If we let θ denote the angle with respect to the source's frame and θ denotes the angle with respect to the observer's frame, then we have

$$\frac{v_0\sqrt{1 - \frac{u^2}{c^2}}}{1 + \frac{u}{c}\cos\theta'} = \mathcal{W}_0(1 - \frac{u}{c}\cos\theta)$$

or

$$1 + \frac{u}{c}\cos\theta' = \frac{1 - \frac{u^2}{c^2}}{1 - \frac{u}{c}\cos\theta} \quad \text{i.e.}$$

$$\cos\theta' = \frac{\cos\theta - \frac{u}{c}}{1 - \frac{u}{c}\cos\theta} \quad (24)$$

Eq. (24) describes the aberration of light. Star aberration arises because the observe moves with the orbital speed of the Earth. If, as SRT asserts, the movement of the light source is equivalent to the movement of the observer, star aberration has to arise as in the case when the source moves. However, the observations of binary stars prove that there is no aberration when the stars move.

In conclusion, due to the Eqs. (22a) and (22b) together, the RDE formula for a moving observer can also be written in the form used for a moving source. Certainly, the formula (22a), which does not have an equivalent in the SRT, is very significant for the formula (22b) for the equality of the longitudinal relativistic Doppler effect. That means the longitudinal relativistic Doppler effect for a moving observer can be also written in the form used for a moving source. The kinematical effect derived by LT is time dilation but not time contraction. Thus, the asymmetry of the kinematical time dilation effect derived by LT makes it difficult to reconcile LT effects and RDE completely. These contradictions cannot be removed without excluding LT and deriving RDE and TRD formulas without LT [3,4]. The inherent asymmetry of kinematical time dilation effect were found also recently by V. Sokolov and G. Sokolov [10,11]. Star aberration and the TDR arise as the result of using the Lorentz force and relativity principle. Both effects do not prove that time dilation takes place in moving systems. Both these phenomena contradict SRT and prove it is false.

References

1.   **Einstein, A**.1905. On the electrodynamics of moving bodies. *Ann. Phys*. 17:891.
2.   **Ferber, R.** 1996. A Missing Link: What is behind de Broglie's periodic henomenon? *Foundations of Physics Letters* 9**:**575-586.

3. **Hamdan, N**. 2005. Derivation of the rlativistic Doppler effect from the Lorentz force. *Apeiron* 12:47-61.

4. **Hamdan**, **N.** 2006. On the Interpretation of the Doppler effect in special relativity (SRT). *Galilean Electrodynamics* 17:29-34**.**

5. **Baird, E.** 2000. Transverse red shift effects without special relativity. <http : //xxx.lanl. gov/abs/physics/ 0010074>.

6. **Parshin, P.** 2001. Theoretical and experimental investigation of the relativistic Doppler effect. *GED* 12:19-20.

7. **Hay, H.J.** 1960. Measurement of the Red shift in an accelerated system using the Mossbauer effect in F57. *Phys. Rev. Letters* 4:165-166.

8. **Ives, H.E. and Stilwell, G.R.** 1938. An experimental study of the rate of a moving atomic clock. *J. Opt. Soc. Am*. 28:215-226.

9. **Ives, H.E. and Stilwell, G.R.** 1941. An experimental study of the rate of a moving atomic clock II. *J. Opt. Soc. Am.* 31:369-374.

10. **Bradley, J.** 1728. Account of a new discovered motion of the fixed stars. *Phil. Trans*. 35:637.

11. **Sokolov, V. and Sokolov, G.** 2005. The Doppler effect and the cosmological red shift. *General Science Journal*, July 20, <www.wbabin.net>.

12. **Sokolov, V. and Sokolov, G.** 2005. The theory of Relativity and Physical Reality. *General Science Journal*, December18, <www.wbabin.net>.